PRECALCULUS

Number and Quantity

- **Vector and Matrix Quantities**
- **Complex Number System**

Algebra

- **Reasoning With Equations and Inequalities**

Functions

- **Interpreting Functions**
- **Building Functions**
- **Trigonometric Functions**

Geometry

- **Geometric Measurement and Dimension**
- **Expressing Geometric Properties with Equations**

Statistics

- **Conditional Probability and the Rules of Probability**

MATHEMATICAL PRACTICES

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

Strand: MATHEMATICAL PRACTICES (MP)

The Standards for Mathematical Practice in PreCalculus describe mathematical habits of mind that teachers should seek to develop in their students. Students become mathematically proficient in engaging with mathematical content and concepts as they learn, experience, and apply these skills and attitudes **(Standards MP.1–8).**

- **Standard P.MP.1   Make sense of problems and persevere in solving them.** Explain the meaning of a problem and look for entry points to its solution. Analyze givens, constraints, relationships, and goals. Make conjectures about the form and meaning of the solution, plan a solution pathway, and continually monitor progress asking, "Does this make sense?" Consider analogous problems, make connections between multiple representations, identify the correspondence between different approaches, look for trends, and transform algebraic expressions to highlight meaningful mathematics. Check answers to problems using a different method.

- **Standard P.MP.2   Reason abstractly and quantitatively.** Make sense of the quantities and their relationships in problem situations. Translate between context and algebraic representations by contextualizing and decontextualizing quantitative relationships. This includes the ability to decontextualize a given situation, representing it algebraically and manipulating symbols fluently as well as the ability to contextualize algebraic representations to make sense of the problem.

- **Standard P.MP.3   Construct viable arguments and critique the reasoning of others.** Understand and use stated assumptions, definitions, and previously established results in constructing arguments. Make conjectures and build a logical progression of statements to explore the truth of their conjectures. Justify conclusions and communicate them to others. Respond to the arguments of others by listening, asking clarifying questions, and critiquing the reasoning of others.

- **Standard P.MP.4   Model with mathematics.** Apply mathematics to solve problems arising in everyday life, society, and the workplace. Make assumptions and approximations, identifying important quantities to construct a mathematical model. Routinely interpret mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

- **Standard P.MP.5   Use appropriate tools strategically.** Consider the available tools and be sufficiently familiar with them to make sound decisions about when each tool might be helpful, recognizing both the insight to be gained as well as the limitations. Identify relevant external mathematical resources and use them to pose or solve problems. Use tools to explore and deepen their understanding of concepts.

- **Standard P.MP.6   Attend to precision.** Communicate precisely to others. Use explicit definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. Specify units of measure and label axes to clarify the correspondence with quantities in a problem. Calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.

■ **Standard P.MP.7   Look for and make use of structure.** Look closely at mathematical relationships to identify the underlying structure by recognizing a simple structure within a more complicated structure. See complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. *For example, see 5 – 3(x – y)² as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers* x *and* y.

■ **Standard P.MP.8   Look for and express regularity in repeated reasoning.** Notice if reasoning is repeated, and look for both generalizations and shortcuts. Evaluate the reasonableness of intermediate results by maintaining oversight of the process while attending to the details.

Strand: NUMBER AND QUANTITY—Vector and Matrix Quantities (N.VM)

Represent and model with vector quantities **(Standards 1–3).** Perform operations on vectors **(Standards 4–5).** Perform operations on matrices and use matrices in applications **(Standards 6–13).**

■ **Standard N.VM.1** Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., *v, |v|, ||v||, v*).

■ **Standard N.VM.2** Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

■ **Standard N.VM.3** Solve problems involving velocity and other quantities that can be represented by vectors.

■ **Standard N.VM. 4** Add and subtract vectors.

   a. Add vectors end to end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

   b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

   c. Understand vector subtraction *v – w as v + (–w)*, where *–w* is the additive inverse of *w*, with the same magnitude as *w* and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

■ **Standard N.VM.5** Multiply a vector by a scalar.

   a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., *as c(vx , vy) = (cvx, cvy).*

b. Compute the magnitude of a scalar multiple cv using $||cv|| = |c|v$. Compute the direction of cv knowing that when $|c|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against vs (for $c < 0$).

■ **Standard N.VM.6** Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

■ **Standard N.VM.7** Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

■ **Standard N.VM.8** Add, subtract, and multiply matrices of appropriate dimensions.

■ **Standard N.VM.9** Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

■ **Standard N.VM.10** Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

■ **Standard N.VM.11** Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

■ **Standard N.VM.12** Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

■ **Standard N.VM.13** Solve systems of linear equations up to three variables using matrix row reduction.

Strand: NUMBER AND QUANTITY—Complex Number Systems (N.CN)

Perform arithmetic operations with complex numbers **(Standard 3).** Represent complex numbers and their operations on the complex plane **(Standards 4–6).** Use complex numbers in polynomial identities and equations **(Standard 10).**

■ **Standard N.CN.3** Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

■ **Standard N.CN.4** Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

■ **Standard N.CN.5** Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example, $(-1 + \sqrt{3}\,i)^3 = 8$, because $(-1 + \sqrt{3}\,i)$ has modulus 2 and argument 120°.*

■ **Standard N.CN.6** Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

■ **Standard N.CN.10** Multiply complex numbers in polar form and use DeMoivre's Theorem to find roots of complex numbers.

Strand: ALGEBRA: REASONING WITH EQUATIONS AND INEQUALITIES (A.REI)

Solve systems of equations **(Standards 8–9).**

8. Represent a system of linear equations as a single matrix equation in a vector variable.

9. Find the inverse of a matrix, if it exists, and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater).

Strand: FUNCTIONS—Interpreting Functions (F.IF)

Analyze functions using different representations **(Standard 7, 10–11).**

■ **Standard F.IF.7** Graph functions expressed symbolically, and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

d. Graph rational functions, identifying zeros, asymptotes, and point discontinuities when suitable factorizations are available, and showing end behavior.

f. Define a curve parametrically and draw its graph.

■ **Standard F.IF.10** Use sigma notation to represent the sum of a finite arithmetic or geometric series.

■ **Standard F.IF.11** Represent series algebraically, graphically, and numerically.

Strand: FUNCTIONS—Building Functions (F.BF)

Build a function that models a relationship between two quantities **(Standard 1).** Build new functions from existing functions **(Standard 4–5).**

■ **Standard F.BF.1** Write a function that describes a relationship between two quantities.

c. Compose functions. *For example, if* T(y) *is the temperature in the atmosphere as a function of height, and* h(t) *is the height of a weather balloon as a function of time, then* T(h(t)) *is the temperature at the location of the weather balloon as a function of time.*

■ **Standard F.BF.4** Find inverse functions.

b. Verify by composition that one function is the inverse of another.

    c. Read values of an inverse function from a graph or a table, given that the function has an inverse.

    d. Produce an invertible function from a non-invertible function by restricting the domain.

■ **Standard F.BF.5** Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Strand: FUNCTIONS—Trigonometric Functions (F.TF)

Extend the domain of trigonometric functions using the unit circle **(Standard 4).** Model periodic phenomena with trigonometric functions **(Standard 6–7).** Prove and apply trigonometric identities **(Standard 9).**

■ **Standard F.TF.4** Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

■ **Standard F.TF.6** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

■ **Standard F.TF.7** Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context

■ **Standard F.TF.9** Prove the addition and subtraction formulas for sine, cosine, and tangent, and use them to solve problems.

Strand: GEOMETRY—Geometric Measurement and Dimension (G.GMD)

Explain volume formulas and use them to solve problems **(Standard 2).**

■ **Standard G.GMD.2** Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

Strand: GEOMETRY—Expressing Geometric Properties With Equations (G.GPE)

Translate between the geometric description and the equation for a conic section **(Standards 2–3).**

■ **Standard G.GPE.2** Derive the equation of a parabola given a focus and a directrix.

■ **Standard G.GPE.3** Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Strand: STATISTICS—Conditional Probability and the Rules of Probability (S.CP)

Understand independence and conditional probability and use them to interpret data **(Standards 2–3).** Use the rules of probability to compute probabilities of compound events in a uniform probability model **(Standards 7–9).**

- **Standard S.CP.2** Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

- **Standard S.CP.3** Understand the conditional probability of *A* given *B* as *P(A and B)/P(B)*, and interpret independence of *A* and *B* as saying that the conditional probability of *B* given *A* is the same as the probability of *B*.

- **Standard S.CP.7** Apply the Addition Rule, *P(A or B) = P(A) + P(B) – P(A and B)*, and interpret the answer in terms of the model.

- **Standard S.CP.8** Apply the general Multiplication Rule in a uniform probability model, *P(A andB) = P(A)P(B|A) = P(B)P(A|B)*, and interpret the answer in terms of the model.

- **Standard S.CP.9** Use permutations and combinations to compute probabilities of compound events and solve problems.