Mathematics Standards

Standards for Mathematical Practice

Make sense of problems and persevere in solving them. (MP.1)

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Less experienced students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Reason abstractly and quantitatively. (MP.2)

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Construct viable arguments and critique the reasoning of others. (MP.3)

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Less experienced students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later. Later, students learn to determine domains to which an argument applies. Students at all levels can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Model with mathematics. (MP.4)

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. This might be as simple as writing an addition equation to describe a situation. A student might apply proportional reasoning to plan a school event or analyze a problem in the community. A student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Use appropriate tools strategically. (MP.5)

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Attend to precision. (MP.6)

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. Less experienced students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Look for and make use of structure. (MP.7)

Mathematically proficient students look closely to discern a pattern or structure. Students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Look for and express regularity in repeated reasoning. (MP.8)

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Early on, students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1)$, $(x-1)(x^2+x+1)$, and $(x-1)(x^3+x^2+x+1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Mathematics Standards Level A

Level A focuses almost entirely on counting, cardinality, number sense, and base-ten operations. This includes developing an understanding of whole number relationships and two-digit place value, as well as strategies for (and fluency with) addition and subtraction. To provide a foundation for algebra, standards introduce the concept of an equation, a variable, and the meaning of the equal sign, all within the context of addition and subtraction within 20. In addition to number, some attention is given to describing and reasoning about geometric shapes in space as a basis for understanding the properties of congruence, similarity, and symmetry, and developing an understanding of linear measurement (length).

LEVEL A (K-1)

Number and Operations: Base Ten
Understand place value. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a. 10 can be thought of as a bundle of ten ones — called a “ten.” b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). (1.NBT.2) Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <. (1.NBT.3)
Use place value understanding and the properties of operations to add and subtract. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. (1.NBT.4) Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. (1.NBT.5) Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. (1.NBT.6)

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. (1.OA.2)

Understand and apply properties of operations and the relationship between addition and subtraction.

Apply properties of operations as strategies to add and subtract. *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)* (1.OA.3)

Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.* (1.OA.4)

Add and subtract with 20.

Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). (1.OA.5)

Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). (1.OA.6)

Work with addition and subtraction.

Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.* (1.OA.7)

Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = □ – 3, 6 + 6 = □* (1.OA.8)

Geometry

Analyze, compare, create, compose shapes.

Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/ "corners") and other attributes (e.g., having sides of equal length). (K.G.4)

Reason with shapes and their attributes.

Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.[12] (1.G.2)

Measurement and Data

Measure lengths indirectly and by iterating length units.

Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.* (1.MD.2)

Represent and interpret data.

Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. (1.MD.4)

[12] Students do not need to learn formal names such as "right rectangular prism."

Mathematics Standards Level B

Level B emphasizes understanding base-ten notation (place value for whole numbers to 1000), developing fluency in addition and subtraction (to 3 digits), understanding and exploring strategies for multiplication and division (within 100), and a foundational understanding of fractions. These skills will prepare students for work with rational numbers, ratios, rates, and proportions in subsequent levels. A critical area of focus is on gaining a foundational understanding of fractions and preparing the way for work with rational numbers. In the areas of measurement and geometry, using standard units of measure and developing understanding of the structure of rectangular arrays and areas are priorities, as well as analyzing two-dimensional shapes as a foundation for understanding area, volume, congruence, similarity and symmetry.

LEVEL B (2-3)

Number and Operations: Base Ten

Understand place value.

Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

a. 100 can be thought of as a bundle of ten tens — called a "hundred."

b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). (2.NBT.1)

Count within 1000; skip-count by 5s, 10s, and 100s. (2.NBT.2)

Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. (2.NBT.3)

Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons. (2.NBT.4)

Use place value understanding and properties of operations to add and subtract.

Add up to four two-digit numbers using strategies based on place value and properties of operations. (2.NBT.6)

Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. (2.NBT.7)

Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900. (2.NBT.8)

Explain why addition and subtraction strategies work, using place value and the properties of operations. (2.NBT.9)

Use place value understanding and properties of operations to perform multi-digit arithmetic.[13]

Use place value understanding to round whole numbers to the nearest 10 or 100. (3.NBT.1)

Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (3.NBT.2)

Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations. (3.NBT.3)

Number and Operations: Fractions[14]

Develop understanding of fractions as numbers.

Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a/b* as the quantity formed by a parts of size 1/*b*. (3.NF.1)

Understand a fraction as a number on the number line; represent fractions on a number line diagram. (3.NF.2)

- Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. (3.NF.2a)
- Represent a fraction *a/b* on a number line diagram by marking off a lengths 1/*b* from 0. Recognize that the resulting interval has size *a/b* and that its endpoint locates the number *a/b* on the number line. (3.NF.2b)

Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. (3.NF.3)

- Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. (3.NF.3a)
- Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model. (3.NF.3b)
- Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.* (3.NF.3c)
- Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. (3.NF.3d)

[13] A range of algorithms may be used.
[14] Expectations at this level in this domain are limited to fractions with denominators 2, 3, 4, 6, 8.

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (2.OA.1)

Add and subtract with 20.

Fluently add and subtract within 20 using mental strategies. Know from memory all sums of two one-digit numbers. (2.OA.2)

Represent and solve problems involving multiplication and division.

Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.* (3.OA.1)

Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8*. (3.OA.2)

Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (3.OA.3)

Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = □ ÷ 3, 6 × 6 = ?*. (3.OA.4)

Understand properties of multiplication and the relationship between multiplication and division.

Apply properties of operations as strategies to multiply and divide.[15] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* (3.OA.5)

Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.* (3.OA.6)

[15] Students need not use formal terms for these properties.

Multiply and divide within 100.

Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. Know from memory all products of two one-digit numbers. (3.OA.7)

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[16] (3.OA.8)

Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.* (3.OA.9)

Geometry

Reason with shapes and their attributes.

Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[17] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. (2.G.1)

Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves, thirds, half of, a third of,* etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape. (2.G.3)

Reason with shapes and their attributes.

Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. (3.G.1)

Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.* (3.G.2)

Measurement and Data

Measure and estimate lengths in standard units.

Measure the length of an object twice, using length units of different lengths for the two

[16] This standard is limited to problems posed with whole numbers having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).
[17] Sizes are compared directly or visually, not compared by measuring.

measurements; describe how the two measurements relate to the size of the unit chosen. (2.MD.2) Estimate lengths using units of inches, feet, centimeters, and meters. (2.MD.3) Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. (2.MD.4)
Relate addition and subtraction to length. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram. (2.MD.6)
Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram. (3.MD.1) Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[18] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[19] (3.MD.2)
Represent and interpret data. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph. (2.MD.10) Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.* (3.MD.3) Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters. (3.MD.4)
Geometric measurement: understand concepts of area and relate to area of multiplication and addition. Recognize area as an attribute of plane figures and understand concepts of area measurement.

[18] Excludes compound units such as cm3 and finding geometric volume of a container.
[19] Excludes multiplicative comparison problems (problems involving notions of “times as much”).

a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.

b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. (3.MD.5)

Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). (3.MD.6)

Relate area to the operations of multiplication and addition. (3.MD.7)

- Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. (3.MD.7a)
- Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. (3.MD.7b)
- Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. (3.MD.7c)
- Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. (3.MD.7d)

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters. (3.MD.8)

Mathematics Standards Level C

More than any other, Level C provides the foundation for all future mathematical studies. Fluency with multidigit whole and decimal numbers as well as calculations with fractions (and the relationships between them) carry the most weight at this level. This extends to working with the concept of ratio and rates, addition and subtraction of fractions, and understanding why the procedures for multiplying and dividing fractions make sense. While the greatest emphasis is still on standards for numbers and operations, attention to algebra and geometry increases considerably in Level C. Reading, writing, and interpreting expressions and equations and generating patterns in numbers and shapes provide a conceptual foundation for functions. In addition, analyzing geometric properties, such as parallelism, perpendicularity, and symmetry, and developing and finding volumes of right rectangular prisms take precedence. Level C also emphasizes sampling techniques and data collection through statistical questioning; to previous standards about data, it adds the understanding of measures of center and spread and display of collected data with line plots.

LEVEL C (4-5, +6)

Number and Operations: Base Ten (+ The Number System)
Generalize place value understanding for multi-digit whole numbers. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.* (4.NBT.1) Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. (4.NBT.2) Use place value understanding to round multi-digit whole numbers to any place. (4.NBT.3)
Use place value understanding and properties of operations to perform multi-digit arithmetic. Fluently add and subtract multi-digit whole numbers using the standard algorithm. (4.NBT.4) Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (4.NBT.5) Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (4.NBT.6)

Understand the place value system.

Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. (5.NBT.1)

Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. (5.NBT.2)

Read, write, and compare decimals to thousandths. (5.NBT.3)

- Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$. (5.NBT.3a)
- Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. (5.NBT.3b)

Use place value understanding to round decimals to any place. (5.NBT.4)

Perform operations with multi-digit whole numbers and with decimals to hundredths.

Fluently multiply multi-digit whole numbers using the standard algorithm. (5.NBT.5)

Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (5.NBT.6)

Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. (5.NBT.7) [Note from panel: Applications involving financial literacy should be used.]

The Number System

Compute fluently with multi-digit numbers and find common factors and multiples.

Fluently divide multi-digit numbers using the standard algorithm. (6.NS.2)

Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. (6.NS.3)

Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).* (6.NS.4)

Number and Operations: Fractions[20]

Extend understanding of fraction equivalence and ordering.

Explain why *a* fraction *a*/*b* is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. (4.NF.1)

Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. (4.NF.2)

Build fractions from unit fractions by applying and extending previous understanding of operations on whole numbers.

Understand a fraction *a*/*b* with $a > 1$ as a sum of fractions 1/*b*. (4.NF.3)

- Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. (4.NF.3a)
- Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* (4.NF.3b)
- Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. (4.NF.3c)
- Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. (4.NF.3d)

Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. (4.NF.4)

- Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).* (4.NF.4a)
- Understand a multiple of *a*/*b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)* (4.NF.4b)
- Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?* (4.NF.4c)

[20] Expectations at this level in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Understand decimal notation for fractions, and compare decimal fractions.

Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.* (4.NF.6)

Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model. (4.NF.7)

Use equivalent fractions as strategy to add and subtract fractions.

Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)* (5.NF.1)

Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.* (5.NF.2)

Apply and extend previous understanding of multiplication and division to multiply and divide fractions.

Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* (5.NF.3)

Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. (5.NF.4)

Interpret multiplication as scaling (resizing), by:

a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1. (5.NF.5)

Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. (5.NF.6)

Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (5.NF.7)

- Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.* (5.NF.7a)
- Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.* (5.NF.7b)
- Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?* (5.NF.7c)

The Number System

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?* (6.NS.1)

Ratios and Proportional Relationships

Understand ratio concepts and use ratio reasoning to solve problems.

Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* (6.RP.1)

Understand the concept of a unit rate *a*/*b* associated with a ratio *a*:*b* with b ≠ 0, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[21] (6.RP.2)

[21] Expectations for unit rates at this level are limited to non-complex fractions.

Operations and Algebraic Thinking

Use the four operations with whole numbers to solve problems.

Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. (4.OA.1)

Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (4.OA.2)

Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (4.OA.3)

Gain familiarity with factors and multiples.

Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. (4.OA.4)

Generate and analyze patterns.

Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.* (4.OA.5)

Write and interpret numerical expressions.

Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. (5.OA.1)

Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as $2 \times (8 + 7)$. Recognize that $3 \times (2100 + 425)$ is three times as large as the $2100 + 425$, without having to calculate the indicated sum or product.* (5.OA.2)

Expressions and Equations

Apply and extend previous understandings of arithmetic to algebraic expressions.

Write and evaluate numerical expressions involving whole-number exponents. (6.EE.1)

Write, read, and evaluate expressions in which letters stand for numbers. (6.EE.2)

- Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.* (6.EE.2a)
- Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* (6.EE.2b)
- Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6\, s^2$ *to find the volume and surface area of a cube with sides of length* $s = 1/2$. (6.EE.2c)

Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.* (6.EE.3)

Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.* (6.EE.4)

Reason about and solve one-variable equations and inequalities.

Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. (6.EE.5)

Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. (6.EE.6)

Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. (6.EE.7)

Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. (6.EE.8)

Represent and analyze quantitative relationships between dependent and independent variables.

Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.* (6.EE.9)

Geometry

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. (4.G.1)

Graph points on the coordinate plane to solve real-world and mathematical problems.

Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate). (5.G.1)

Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. (5.G.2)

Classify two-dimensional figures into categories based on their properties.

Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.* (5.G.3)

Solve real-world and mathematical problems involving area, surface area, and volume.

Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. (6.G.1)

Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. (6.G.3)

Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems. (6.G.4)

Measurement and Data

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. (4.MD.2)

Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.* (4.MD.3)

Geometric measurement: understand concepts of angle and measure angles.

Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a “one-degree angle,” and can be used to measure angles.
b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. (4.MD.5)

Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. (4.MD.6)

Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. (4.MD.7)

Convert like measurement units within a given measurement system.

Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems. (5.MD.1)

Represent and interpret data.

Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.* (5.MD.2) [Note from panel: Plots of numbers other than measurements also should be encouraged.]

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.

b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. (5.MD.3)

Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. (5.MD.4)

Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. (5.MD.5)

- Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. (5.MD.5a)
- Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. (5.MD.5b)
- Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems. (5.MD.5c)

Statistics and Probability

Develop understanding of statistical variability.

Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.* (6.SP.1)

Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. (6.SP.2)

Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. (6.SP.3)

Summarize and describe distributions.

Display numerical data in plots on a number line, including dot plots, histograms, and box plots. (6.SP.4) [Also see S.ID.1]

Mathematics Standards Level D

Like preceding levels, Level D also emphasizes number sense and operations, but here the attention is on fluency with all four operations with rational numbers—both negative and positive. The foundation for understanding of irrational numbers is built here, including calculation with square and cube roots and solving simple quadratic equations. Another keen area of concentration is algebra and functions: formulating and reasoning about expressions, equations, and inequalities; solving linear equations and systems of linear equations; grasping the concept of a function; and using functions to describe quantitative relationships. Level D is also where understanding and applying ratios, rates, and proportional reasoning—forming a bridge between rational number operations and algebraic relationships—are developed. Building on the geometric analysis in Level C, the focus turns to analyzing two- and three-dimensional figures using distance, angle, similarity, and congruence, and understanding basic right triangle trigonometry. Having worked with measurement data in previous levels, students at this level develop notions of statistical variability and learn to understand summary statistics and distributions. The concept of probability is introduced and developed at this level.

LEVEL D (+6, 7-8)

The Number System

Apply and extend previous understandings of numbers to the system of rational numbers.

Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. (6.NS.5)

Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. (6.NS.6)

- Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite. (6.NS.6a)
- Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. (6.NS.6b)
- Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. (6.NS.6c)

Understand ordering and absolute value of rational numbers. (6.NS.7)

- Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.* (6.NS.7a)

- Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3º C > –7º C to express the fact that –3º C is warmer than –7º C.* (6.NS.7b)
- Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.* (6.NS.7c)
- Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.* (6.NS.7d)

Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. (6.NS.8)

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. (7.NS.1)

- Describe situations in which opposite quantities combine to make 0. *For example, if a check is written for the same amount as a deposit, made to the same checking account, the result is a zero increase or decrease in the account balance.* (7.NS.1a)
- Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. (7.NS.1b)
- Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. (7.NS.1c)
- Apply properties of operations as strategies to add and subtract rational numbers. (7.NS.1d)

Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. (7.NS.2)

- Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. (7.NS.2a)
- Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $–(p/q) = (–p)/q = p/(–q)$. Interpret quotients of rational numbers by describing real-world contexts. (7.NS.2b)

- Apply properties of operations as strategies to multiply and divide rational numbers. (7.NS.2c)
- Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. (7.NS.2d)

Solve real-world and mathematical problems involving the four operations with rational numbers. (7.NS.3)

Know that there are numbers that are not rational, and approximate them by rational numbers.

Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of* $\sqrt{2}$, *show that* $\sqrt{2}$ *is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations*. (8.NS.2)

Understand ratio concepts and use ratio reasoning to solve problems.

Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. (6.RP.3)

- Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. (6.RP.3a)
- Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* (6.RP.3b)
- Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. (6.RP.3c)
- Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. (6.RP.3d)

Analyze proportional relationships and use them to solve real-world and mathematical problems.

Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction* $^{1/2}/_{1/4}$ *miles per hour, equivalently 2 miles per hour.* (7.RP.1)

Recognize and represent proportional relationships between quantities. (7.RP.2)

- Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. (7.RP.2a)
- Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. (7.RP.2b) [Also see 8.EE.5]

- Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.* (7.RP.2c)
- Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r)* where *r* is the unit rate. (7.RP.2d)

Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.* (7.RP.3) [Also see 7.G.1 and G.MG.2]

Expressions and Equations

Use properties of operations to generate equivalent expressions.

Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. (7.EE.1)

Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."* (7.EE.2) [Also see A.SSE.2, A.SSE.3, A.SSE.3a, A.CED.4]

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.* (7.EE.3)

Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. (7.EE.4) [Also see A.CED.1 and A.REI.3]

- Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* (7.EE.4a) [Also see A.CED.1 and A.REI.3]
- Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.* (7.EE.4b) [Also see A.CED.1 and A.REI.3]

Work with radicals and integer exponents.

Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{(-5)} = 3^{(-3)} = (1/3)^3 = 1/27$. (8.EE.1) [Also see F.IF.8b]

Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. (8.EE.2) [Also see A.REI.2]

Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.* (8.EE.3)

Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology. (8.EE.4) [Also see N.Q.3]

Understand the connections between proportional relationships, lines, and linear equations.

Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* (8.EE.5) [Also see 7.RP.2b]

Analyze and solve linear equations and pairs of simultaneous linear equations.

Solve linear equations in one variable. (8.EE.7) [Also see A.REI.3]

- Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). (8.EE.7a)
- Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. (8.EE.7b)

Analyze and solve pairs of simultaneous linear equations. (8.EE.8)

- Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. (8.EE.8a)
- Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,* $3x + 2y = 5$ *and* $3x + 2y = 6$ *have no solution because* $3x + 2y$ *cannot simultaneously be 5 and 6.* (8.EE.8b) [Also see A.REI.6]
- Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.* (8.EE.8c)

Functions

Define, evaluate, and compare functions.

Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[22] (8.F.1) [Also see F.IF.1]

Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.* (8.F.3)

Use functions to model relationships between quantities.

Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. (8.F.4) [Also see F.BF.1 and F.LE.5]

Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. (8.F.5) [Also see A.REI.10 and F.IF.7]

Geometry

Draw, construct, and describe geometrical figures and describe the relationships between them.

Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. (7.G.1) [Also see 7.RP.3]

Solve real-life and mathematical problems involving angle, measure, area, surface area, and volume.

Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. (7.G.4)

Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. (7.G.5)

Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. (7.G.6) [Also see G.GMD.3]

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[22] Function notation is not required at this level.

Understand congruence and similarity using physical models, transparencies, or geometry software.

Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. (8.G.2) [Also see G.SRT.5]

Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. (8.G.4) [Also see G.SRT.5]

Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.* (8.G.5)

Understand and apply the Pythagorean Theorem.

Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. (8.G.7)

Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. (8.G.8)

Statistics and Probability

Summarize and describe distributions.

Summarize numerical data sets in relation to their context, such as by:

a. Reporting the number of observations.
b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. (6.SP.5)

Use random sampling to draw inferences about a population.

Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. (7.SP.1)

Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling* *words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.* (7.SP.2)

Draw informal comparative inferences about two populations.

Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.* (7.SP.3)

Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in one chapter of a science book are generally longer or shorter than the words in another chapter of a lower level science book.* (7.SP.4) [Also see S.ID.3]

Investigate chance processes and develop, use, and evaluate probability models.

Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. (7.SP.5)

Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.* (7.SP.6)

Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. (7.SP.7)

- Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* (7.SP.7a)
- Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?* (7.SP.7b)

Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. (7.SP.8a)

Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event. (7.SP.8b)

Investigate patterns of association in bivariate data.

Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. (8.SP.1) [Also see S.ID.1]

Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. (8.SP.2)

Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* (8.SP.3) [Also see S.ID.7]

Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they like to cook and whether they participate actively in a sport. Is there evidence that those who like to cook also tend to play sports?* (8.SP.4) [Also see S.ID.5]

Mathematics Standards Level E

Themes introduced and developed in earlier levels continue and deepen in Level E. Having already extended arithmetic calculations from whole numbers to fractions and from fractions to rational and irrational numbers, understanding the real number system comes to the fore. Understanding radical expressions, using and interpreting units in problem solving, and attending to precision are important areas of focus. Prior work with proportional relationships and functions expands from linear expressions, equations, and functions to quadratic, rational, exponential, and polynomial. To bridge the gap between algebra and geometry, rates and relationships are applied to density models. Work also advances in geometry, including using congruence and similarity criteria to prove relationships in geometric figures and determining volumes of cylinders, pyramids, cones, and spheres. Basic skills and knowledge of statistics and probability are applied in a modeling context, in which students interpret and compare data distributions and understand issues of correlation and causation.

Note: Making mathematical models is a Standard for Mathematical Practice (MP.4), and specific modeling standards appear throughout the high school standards indicated by an asterisk (*).

LEVEL E (High School)

Number and Quantity: The Real Number System

Extend the properties of exponents to rational exponents.

Rewrite expressions involving radicals and rational exponents using the properties of exponents. (N.RN.2)

Number and Quantity: Quantities

Reason quantitatively and use units to solve problems.

Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.* (N.Q.1)

Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.* (N.Q.3) [Also see 8.EE.4]

Algebra: Seeing Structure in Expressions

Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.* (A.SSE.1)

- Interpret parts of an expression, such as terms, factors, and coefficients.* (A.SSE.1a)

Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$. (A.SSE.2) [Also see 7.EE.2]

Write expressions in equivalent forms to solve problems.

Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* (A.SSE.3) [Also see 7.EE.2]

- Factor a quadratic expression to reveal the zeros of the function it defines.* (A.SSE.3a) [Also see 7.EE.2]

Algebra: Arithmetic with Polynomials and Rational Expressions

Perform arithmetic operations on polynomials.

Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. (A.APR.1) [Note from panel: Emphasis should be on operations with polynomials.]

Rewrite rational expressions.

Rewrite simple rational expressions in different forms; write $^{a(x)}/_{b(x)}$ in the form $q(x) + {}^{r(x)}/_{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. (A.APR.6)

Algebra: Creating Equations

Create equations that describe numbers or relationships.

Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*.* (A.CED.1) [Also see 7.EE.4, 7.EE.4a, and 7.EE.4b]

Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.* (A.CED.2)

Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non- viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.** (A.CED.3)

Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law* $V = IR$ *to highlight resistance* R.* (A.CED.4) [Also see 7.EE.2]

Algebra: Reasoning with Equations and Inequalities

Understand solving equations as a process of reasoning and explain the reasoning.

Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. (A.REI.1)

Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. (A.REI.2) [Also see 8.EE.2]

Solve equations and inequalities in one variable.

Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. (A.REI.3) [Also see 7.EE.4, 7.EE.4a, 7.EE.4b, and 8.EE.7]

Solve quadratic equations in one variable. (A.REI.4)

Solve systems of equations.

Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. (A.REI.6) [Also see 8.EE.8b]

Represent and solve equations and inequalities graphically.

Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). (A.REI.10) [Also see 8.F.5]

Functions: Interpreting Functions

Understand the concept of a function and use function notation.

Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. (F.IF.1) [Also see 8.F.1]

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. (F.IF.2)

Interpret functions that arise in applications in terms of the context.

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *For example, for a quadratic function modeling a projectile in motion, interpret the intercepts and the vertex of the function in the context of the problem.** (F.IF.4) [*Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.]*

Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble* *n engines in a factory, then the positive integers would be an appropriate domain for the function.** (F.IF.5)

Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.* (F.IF.6) [NOTE: See conceptual modeling categories.]

Analyze functions using different representations.

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* (F.IF.7) [Also see 8.F.5]

Use properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in an exponential function and then classify it as representing exponential growth or decay.* (F.IF.8b) [Also see 8.EE.1]

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* (F.IF.9)

Functions: Building Functions

Build a function that models a relationship between two quantities.

Write a function that describes a relationship between two quantities.* (F.BF.1) [Also see 8.F.4]

Functions: Linear, Quadratic, and Exponential Models

Construct and compare linear, quadratic, and exponential models and solve problems.

Distinguish between situations that can be modeled with linear functions and with exponential functions.* (F.LE.1)

- Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.* (F.LE.1b)
- Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.* (F.LE.1c)

Interpret expressions for functions in terms of the situation they model.

Interpret the parameters in a linear or exponential function in terms of a context.* (F.LE.5) [Also see 8.F.4]

Geometry: Congruence

Experiment with transformations in the plane.

Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. (G.CO.1)

Geometry: Similarity, Right Triangles, and Trigonometry

Prove theorems involving similarity.

Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. (G.SRT.5) [Also see 8.G.2 and 8.G.4]

Geometry: Geometric Measurement and Dimension

Explain volume formulas and use them to solve problems.

Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.* (G.GMD.3) [Also see 7.G.6]

Geometry: Modeling with Geometry

Apply geometric concepts in modeling situations.

Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).* (G.MG.2) [Also see 7.RP.3]

Statistics and Probability: Interpreting Categorical and Quantitative Data

Summarize, represent, and interpret data on a single count or measurable variable.

Represent data with plots on the real number line (dot plots, histograms, and box plots). (S.ID.1) [Also see 6.SP.4 and 8.SP.1]

Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). (S.ID.3) [Also see 7.SP.4]

Summarize, represent, and interpret data on two categorical and quantitative variables.

Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. (S.ID.5) [Also see 8.SP.4]

Interpret linear models.

Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. (S.ID.7) [Also see 8.SP.3]

Distinguish between correlation and causation. (S.ID.9)